Q.P. Code: 18HS0830 **R18**

**Reg. No:** | | | | | | | | | | |

**SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR**

(AUTONOMOUS)

**B.Tech I Year I Semester Supplementary Examinations December-2021**

**MATHEMATICS-I**

(Common to ALL)

Time: 3 hours Max. Marks: 60

**PART-A**

(Answer all the Questions **5 x 2 = 10** Marks)

1 a Find the Eigen values of the matrix A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **2M**

b Prove that $\Gamma(1) = 1$. **2M**

c Define Total differential coefficient. **2M**

d Test for convergence the series $\sum \frac{n^3}{3^n}$. **2M**

e Find Fourier constant $a_0$ for $f(x) = 1 - x^2$ in [-1,1]. **2M**

**PART-B**

(Answer all Five Units 5 x 10 = 50 Marks)

**UNIT-I**

2 a Find the rank of the matrix A= $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$. **5M**

b Determine the Eigen values of $A^{-1}$ *where* A= $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. **5M**

**OR**

3 Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$ into the canonical form by Orthogonal transformation. **10M**

**UNIT-II**

4 a Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem. **5M**

b Expand $\log_e x$ in power of $(x-1)$ and hence evaluate log 1.1 correct to 4 decimal places using Taylor's theorem. **5M**

**OR**

5 a Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}}\, dx = \frac{1}{5} B\left(\frac{2}{5}, \frac{1}{2}\right)$. **5M**

b Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}}\, dx = \frac{1}{2}\, \beta\left(1, \frac{1}{2}\right)$. **5M**

**UNIT-III**

6 a If $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3$ *, then show that* $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$. **5M**

b If $u = x^2 + y^2 + z^2\ and\ x = e^{2t}, y = e^{2t}cos3t, z = e^{2t}sin3t, find\ \frac{du}{dt} = ?$ **5M**

**OR**

7 a Find a point on the plane $3x + 2y + z - 12 = 0$, , which is nearest to the origin. **6M**

b Find the shortest and longest distance from the point$(3,1,-1)$ $to\ the\ sphere\ x^2+y^2+z^2=4$. **4M**

**UNIT-IV**

8 Examine the following sequences for convergence: **10M**

$i)\ a_n = \frac{n^2-2n}{3n^2+n}$ $\quad ii)\ a_n = 3 + (-1)^n$.

**OR**

9 State the value of $x$, for which the following series converge: **10M**

i) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - - - - - - - - \infty$,

ii) $\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + - - - - - - - \infty$.

**UNIT-V**

10 Find the Fourier series to represent the function $f(x) = x^2$ for $-\pi < x < \pi$ and hence show that **10M**

(i)$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - - - - - = \frac{\pi^2}{12}$. (ii)$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - - - - = \frac{\pi^2}{6}$.

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} - - - - = \frac{\pi^2}{8}$

**OR**

11 a Find the half range sine series expansion of $f(x) = x^2$ when$0 < x < 4$. **5M**

b Find the half range cosine series expansion of $f(x) = x(2-x)$ in $0 \le x \le 2$. **5M**

***END***